H-MRSTM: A Competitive Search Algorithm for Heterogeneous Group of Robots

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Abstract: **This paper introduces an algorithm for an on-line problem where a group of heterogeneous robots must find a target in an unbounded environment whose geometry is a priori unknown. The performance of the motion planning algorithm is measured by competitiveness, which is the functional relation between the time it took the first robot to find the target and the optimal off-line solution calculated while knowing all the information about the target position and the environment's geometry. Velocity heterogeneity implies that the path lengths of robots with different velocities traveling the same period of time will not be equal. Heterogeneous Multi Robot Search Time Multiplication (H-MRSTM) is developed and its upper bound is found to be quadratic in the optimal off-line solution. The algorithm is developed to ensure completeness, robustness and optimality in the sense of the time to accomplish the task. The new algorithm is tested in simulations.**

Key-Words: **- multi-robot, mobile, heterogeneous, complete algorithm.**

1 Introduction

Mobile robots play a significant role in a vast range of domains. Mobile robots can be found in military and police applications such as armed patrol vehicles, surveillance [12], and bomb disarming robots. In industry mobile robots serve as AGV's conveying and handling materials [3]. Examples for civilian tasks include search and rescue missions and demining [14], space application include planetary exploration and material acquisition on distant planets [5, 4]. Additional applications include service robotics, cleaning household, pool, and lawn mowing.

Exploration missions using short range sensors and target finding problems where the target position is unknown, and the robot can identify it only upon arrival, results in covering a significant area. A well known single robot Spanning Tree Covering algorithm (STC) [8] is a grid based coverage algorithm. STC is proved to be complete and optimal in the sense that the area will be covered only once (i.e. without repetitions). STC can be used either off-line or on-line. STC is designed to work only in bounded environments.

Many grid-based coverage algorithms are derived or compared with STC for performance measure. Agmon et al. [1] introduce a distributed construction of a spanning tree while considering different initial locations of the robots. Zheng et al. [18] rely on STC, too, and introduce a multi robot algorithm which uses heuristics to off-line construct partially overlapping trees. In the notable early paper [11], Kurabayashi et al. present a sweeping algorithm for homogeneous multi robot, which, unlike the above mentioned algorithms, uses exact cell decomposition. Though seems complete and efficient, proofs are not provided and it is only shown through simulations and experiments. The above mentioned multi robot coverage algorithms, are off-line and cannot be used in on-line manner without major revisions. Hazon et al. [9] extended their previous off-line algorithm and created an on-line version of it. As their previous off-line algorithm, the on-line version also focuses on non redundancy of the covered area thus losing optimality in search time.

Papers on multi-robot motion planning usually con-

sider communication between the robots to cooperate and enhance the performance. In [19] self localization is made possible by scattering *RFID* tags which have memory and can communicate within a short radius. Thus, no long range communication is needed. Tagging the environment is also used by Ferranti et al. [7] to overcome wireless communication disorders in indoor environments. Each tag marks a cell either as explored or as visited and consist information like the direction of the parent cell and the robot's ID number. *Line of Sight (LOS)* is a communication and a vision constraint. In [17, 2] the robots rely on LOS for communication and for vision and assume omnidirectional visibility, a requirement which does not prove to be effective in condensed areas, where LOS is usually not maintained. The main drawback of such tagging and marking algorithms is the extra equipment they need to hold and deploy the tags devices, which adds weight and physical size to the robots.

A few coverage algorithm are derived from the Boustrophedon decomposition [6], an exact cellular decomposition developed for a single robot. The cells are created by the natural structure of the obstacles in the environment, and each cell is covered by simple back and forth vertical movements. Rekleitis et al. [13], extended the Boustrophedon decomposition to multiple robots, using two type of robots. This solution relies on line-of-sight communication between the explorers and inside the teams. In a later work, Rekleitis et al. [14], focus on de-mining. The solution demands scattering the robots evenly along the edge of the field to be covered and combines an auction mechanism to divide the uncovered area after the initial run. Kong et al. [10] propose a solution which is based on the Boustrophedon decomposition, but does not require line-of-sight between the robots, instead, communication is assumed to be available without restrictions. All the algorithm that were mentioned in this section are restricted to operate within a bounded environment.

Our research explores the problem of finding a target whose position is unknown, in an a priori unknown, unbounded environment by a group of heterogeneous robots starting from a common location, a problem which was not treated as a whole before. Heterogeneity here is considered as heterogeneity in robots velocity, where each robot travels in different velocity. The robots are totaly autonomous and their onboard setup is composed of short range, e.g., tactile sensors. Communication between the robots is needed only in the beginning and at the end of the execution, permitting a decentralized system. A deterministic, complete, robust and optimal solution is presented.

The structure and contributions of the paper are as follows. In the next section we define the algorithm performance measure, and generalize complexity. In Section 3 we introduce *H-MRSTM*, a new competitive algorithm. In Section 4 we analyze *H-MRSTM* performance and competitiveness. In Section 5 we present simulation results of *H-MRSTM*. Finally we conclude and discuss additional research directions and future work.

2 Performance Measure and Generalized Time Competitiveness

Performance of on-line motion algorithms is usually measured by the path length traveled prior to finding the target. Since robots with varying velocities are employed, each will travel different path length in the same time. Thus, here time is the main performance measure.

We compare the on-line travel time of the robot that finds the target to the off-line optimal travel time solution. The relation between the on-line execution time and the optimal solution is defined as time competitiveness. The generalized definition of time competitiveness is used.

Definition 1 (Generalized Time Competitiveness)**.** *An on-line algorithm solving a task P is $f(T_{opt})$- competitive when T is bounded from above by a scalable function $f(T_{opt})$ over all instances of P. In particular, $T \leq c_1 T_{opt} + c_0$ is the linear time competitiveness, while $T \leq c_2 T_{opt}^2 + c_1 T_{opt} + c_0$ is a quadratic time competitiveness, where the c_i's are positive constant coefficients that depend on the robot size D, the robot's velocity, the number of robots, and the geometry of the environment.*

The meaning of scalability is as follows. When performance is measured in time units such as seconds, one must ensure that both sides of the relationship $T \leq f(T_{opt})$ posses the same units, so that change of scale would not affect the bound. For instance, the coefficient c_2 in the relationship $T \leq c_2 T_{opt}^2 + c_1 T_{opt} + c_0$ must have units of sec^{-1}, c_1 must be unitless, and c_0 must have units of *sec*. Note that the definition of $f(T_{opt})$-competitiveness focuses on a particular algorithm solving the task P. However, the objective is

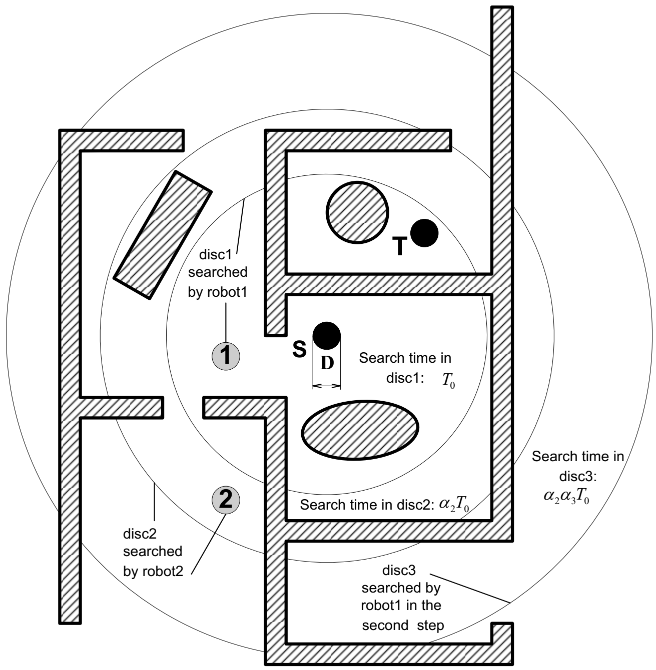

Figure 1: A group of two robots launched by *H-MRSTM* searching for the target.

to characterize the lowest upper bound that can be achieved over all on-line algorithms for P.

3 *H-MRSTM* Search Algorithm

In *H-MRSTM* the robots are heterogeneous in their velocities. *H-MRSTM* deploys each of its robots to search for the target in concentric discs whose areas grows. After a robot finished searching for the target and did not find it, it moves to the next unoccupied search disc to search for the target in it. Eventually, the search disc will contain a path to the target and it will be found.

H-MRSTM algorithm launches multiple robots from a common starting point S and assigns each robot j to a disc to search for the target T in it. All the discs are concentric and S is their center. n robots are deployed, and their velocities are $v_i = \beta_i v,\ \beta_{i+1} \geq \beta_i \geq 1,\ i = 1, \ldots, n$, where v is the velocity of the slowest robot whose assigned index is one. The robots indices are ordered according to their velocity from the slowest to the fastest, and β_i it the ratio between the velocity of the i^{th} robot and the velocity of the slowest robot.

The first robot, R_1 is designated to search in the initial disc. The radius of the first disc is set such that the first robot will complete searching for the target in it within T_0 time. Note that in worst case the robot will have to cover the entire disc and therefor the disc radius is set to $r_0 = \sqrt{T_0 v D}/\sqrt{\pi}$. Each of the following robots starts its search in a disc of radius $r_i = \sqrt{T_i \beta_i v D}/\sqrt{\pi}$, and since the disc area is $T_i \beta_i v D$ the i^{th} robot will complete cover the disc in worst case scenario within time of T_i. We define T_i to be the *search time* of the i^{th} disc. Each robot, after completing covering its disc and not finding the target yet, will move to start search the target in a disc whose search time is larger by a factor of $\alpha_j > 1$ from the previous search disc. Each robot has its own multiplication factor $\alpha_j > 1$ according to its velocity, hence, the search times within the discs will be,

$$T_0, \alpha_2 T_0, \alpha_2\alpha_3 T_0, \alpha_2\alpha_3\alpha_4 T_0, \ldots$$

$$\ldots, \prod_{i=2}^{n} \alpha_i T_0, \prod_{i=1}^{n} \alpha_i T_0, \alpha_2 \prod_{i=1}^{n} \alpha_i T_0, \alpha_2\alpha_3 \prod_{i=1}^{n} \alpha_i T_0, \ldots$$

$$\ldots, \prod_{i=2}^{n} \alpha_i \prod_{i=1}^{n} \alpha_i T_0, \ldots \ .$$

For example, in Figure 1 *H-MRSTM* deploys a group of two robots to search for the target, robot 1 is initially assigned to search for the target inside a disc of search time T_0 and robot R_2 is assigned to search inside a disc of search time $\alpha_2 T_0$. After robot 1 completes covering the entire portion of disc 1 which is accessible from S and fails to find the target, it starts searching for the target inside disc 3 of search time $\alpha_1\alpha_2 T_0$, in this case, robot R_1 will find the target while searching in disc 3, before or after robot R_2 completed searching in disc 2 and moved on to disc 4 of search time $\alpha_1\alpha_2^2 T_0$. Each robot searches for the target in the accessible portion of the disc allocated to it until the target is detected, or until the entire region accessible from S is explored without finding T. The search process in each disc is the same as in *MRSAM*[16, 15] and STC algorithms. A formal description of *H-MRSTM* algorithm appears in Figure 2.

Though each consequent disc contains the previous one, the series of the covering times form a converging series, thus yielding an upper bound on the path length and an optimal solution is obtained. The following conditions formalizes the last idea.

Condition 1. *(Search disc's area ratio) The area of each consequent search disc is greater than the area of the previous search disc.*

Basic *H-MRSTM* Algorithm's Pseudocode

Sensors: A position sensor.
An obstacle detection sensor.
A target detection sensor.

Input: A start point S .
An initial search time T_0.
A group of n searching robots, with different velocities,
$v_i = \beta_i v,\ \beta_{i+1} \geq \beta_i \geq 1,\ \forall i, 1 \leq i < n$

Initialization:
For each robot $R_j,\ j = 1, \ldots, n$:
Set multiplication factor α_j.
Set initial search time $T_{1_{(R_j)}} = T_0 \quad , j = 1$
$T_{1_{(R_j)}} = \prod_{i=2}^{j} \alpha_i T_0,\ j \neq 1$

For each robot j,
Repeat:
Execute a *coverage tour* on the grid contained in the disc of search time T_j centered at $\mathcal{S}$. Scan each new free cell and its partially occupied neighbor cells for $\mathcal{T}$
until one of the following occurs:
(1) The target is reached: STOP.
(2) If no new free cell is encountered during the current coverage tour:
STOP, the target is unreachable.
(3) Else, move to the next unoccupied disc k:
Set $T_{k_{(R_j)}} = (\prod_{i=1}^{n} \alpha_i)^{\left(\frac{k-j}{n}\right)} T_0 \quad , j = 1.$
Set $T_{k_{(R_j)}} = (\prod_{i=1}^{n} \alpha_i)^{\left(\frac{k-j}{n}\right)} \prod_{i=2}^{j} \alpha_i T_0,\ j \neq 1.$

End of Repeat loop

Figure 2: *H-MRSTM* Pseudocode.

During the same period of time, robots with different velocities will cover different areas, and the ratio of the covered areas is the same as the ratio of the velocities. Consequently, a fast robot might finish covering the next search disc before the slow robot finished searching in the previous disc, thus, for *H-MRSTM*, condition 1 does not suffice, and the following condition completes it.

Condition 2. *(Search time ratio) The time of search within each consequent search disc is greater than the time of search within the previous search disc.*

4 *H-MRSTM* Performance

The general definitions and assumptions are as follows. n robots with n different velocities, $v_i = \beta_i v$, $i = 1, 2, \ldots, n$, $\beta_{i+1} \geq \beta_i \geq 1\ \forall i\, 1 \leq i < n$, are deployed. Assuming initial search time T_0, individual multiplication factors $\alpha_i > 1$, $i = 1, 2, \ldots, n$, and generally, the area of the i^{th} disc, $A_i = T_i v_j D$. We generalize $T_{k_{(R_j)}}$, the time it takes for robot j to cover disc number k, in the following way,

$$T_{k_{(R_j)}} = \left(\prod_{i=1}^{n} \alpha_i\right)^{\left(\frac{k-j}{n}\right)} T_0 \qquad , j = 1,$$

$$T_{k_{(R_j)}} = \left(\prod_{i=1}^{n} \alpha_i\right)^{\left(\frac{k-j}{n}\right)} \prod_{i=2}^{j} \alpha_i T_0,\ j \neq 1.$$

Where T_0 is the initial time to search in the first disc, k is the disc or iteration number, j is the robot id number, n is the total number of robots, and $\alpha_i,\ i = 1, \ldots, n$ are the individual multiplication factors.

Since each consequent search time is multiplied by α_i and each $\alpha_i \geq 1$, conditions 2 holds true for *H-MRSTM* with n robots, too. Applying condition 1, $A_i < A_{i+1}$, yields the following results. For each transition from robot R_i to robot R_{i+1}, $1 \leq i < n$, e.g. R_2 to R_3, $\beta_i < \alpha_{i+1}\beta_{i+1}$. For each transition from robot R_i to robot R_1, $i = n$, $\beta_i < \alpha_1\beta_1$.

Next, the performance of *H-MRSTM* is analyzed by calculating the total time it took to find the target. Due to the difference in velocities, n cases are considered here, in each case a different robot finds the target. The following theorem establishes the quadratical functional relation of the time to find the target and the optimal solution.

Theorem 1. *(Quadratic competitive complexity) Assume target* T *is reachable from* S. *Let* H-MRSTM *use* n *robots with velocities:*
$v_i = \beta_i v,\ \forall i,\ i = 1, 2 \ldots n,$
$\beta_{i+1} \geq \beta_i \geq 1,\ \forall i,\ i = 1, 2 \ldots n-1.$
Then the traveling time of robot j *which found the target:*

$$T_{R_j} < \frac{\pi \alpha_j \beta_n^2 v \left(\prod_{i=1}^{n} \alpha_i\right)}{\beta_j D \left(\prod_{i=1}^{n} \alpha_i - 1\right)} T_{opt}^2,$$

$$and,\ \ \alpha_i = \frac{(n+1)^{\left(\frac{1}{n}\right)} \beta_i^{\frac{n-1}{n}}}{\prod_{k=2, k\neq i}^{n} \beta_k^{\frac{1}{n}}}.$$

The proof was relegated to Appendix A.

5 Simulation

The performance analysis of *H-MRSTM* presented in the previous section measures the behavior of the algorithm in worst case scenarios. Simulation analysis

of *H-MRSTM* can test how the algorithm behaves in the average case.

The application was written in Microsoft® Visual Studio .NET C#. Different environments can be drawn, loaded and saved either as text or as bitmap files. The parameters which can be varied and tested in the application are the number of robots and their individual velocities, the robots width, and the initial disc search time. Finally, the target can be placed in various positions. *H-MRSTM* simulation is using an implementation of *STC* as the covering algorithm within each disc. This version of *STC* covers the free area only once by on-line generating a spanning tree of the $2D$ grid using a *DFS* like search method. The robot circumnavigate the spanning tree single time and finishes in the starting position. The *STC* algorithm regards the bounding disc as a virtual obstacle.

Here we show a simulation example where *H-MRSTM* deploys 4 robots to search for the target in an environment lightly populated with obstacles. The velocities of the robots are,
$\beta_1 = 1v, \beta_2 = 1.070v, \beta_3 = 1.144v, \beta_4 = 1.307v$,
and accordingly, the multiplication factors are,
$\alpha_1 = 1.33, \alpha_2 = 1.42, \alpha_3 = 1.52, \alpha_4 = 1.74$. The initial T_0 corresponds to $R_0 = 14D$. The target lies outside the the first four assigned discs and thus none of the robots can reach the target in its first round (Figure 3). The paths of the four robots during their first search discs are depicted in Figures 3,4,5, and 6. In each figure only one robot's path is depicted for clarity reasons. The blue circles mark the discs boundaries, in the simulation presented here the robots path is overriding other robot's discs.

Each robot, when its time comes, move on to search in its next search disc, and, eventually, robot no. 2 finds the target after 1837 D steps. Robots 1,3, and 4 travel 1715, 1963, and 2245 steps respectively. In Figure 7 Robot 2 reaches the target. The optimal off-line path is depicted in Figure 8, and its length is 44 D steps or 32.25 D steps in aerial line. When selecting $v = 1D[m/sec]$, T_2, the time it took for robot 2 to reach the target becomes $T_2 = 1716.8[sec]$ and T_{opt}, the optimal off-line time to reach is $T_{opt} = 24.67$. The large difference between them is due to the fact that searching for a target takes much more time than just traveling to the target when its position is known.

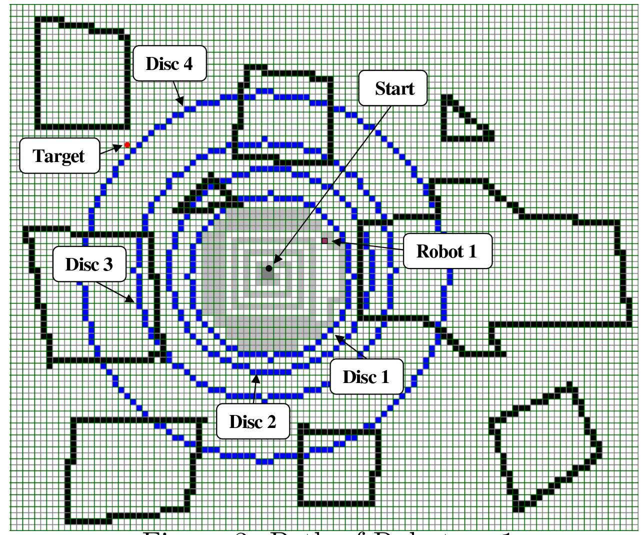

Figure 3: Path of Robot no.1

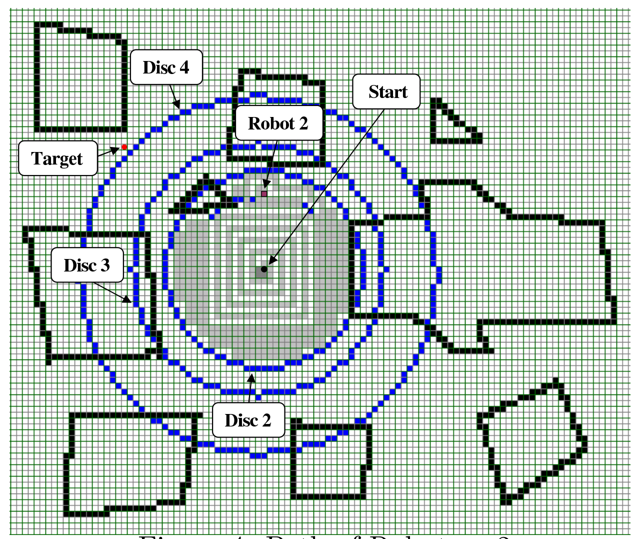

Figure 4: Path of Robot no.2

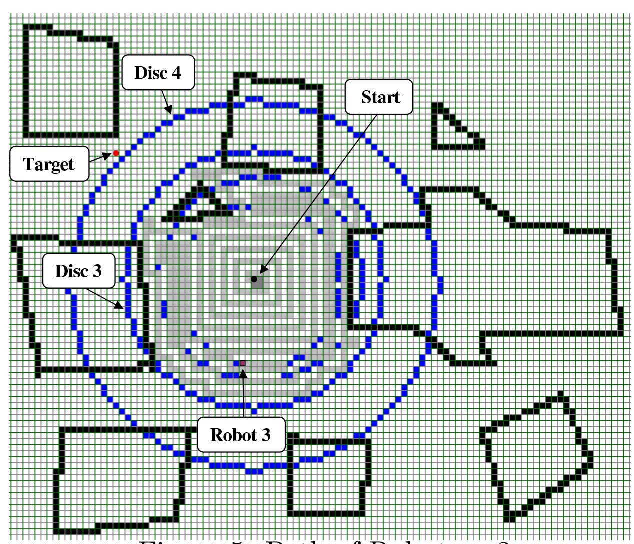

Figure 5: Path of Robot no.3

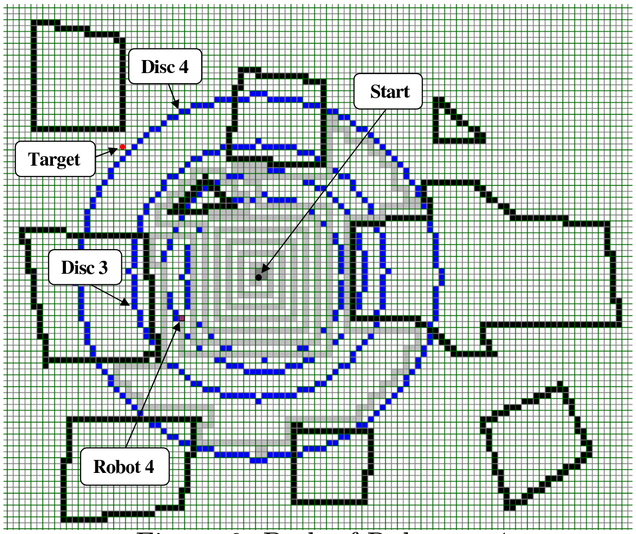

Figure 6: Path of Robot no.4

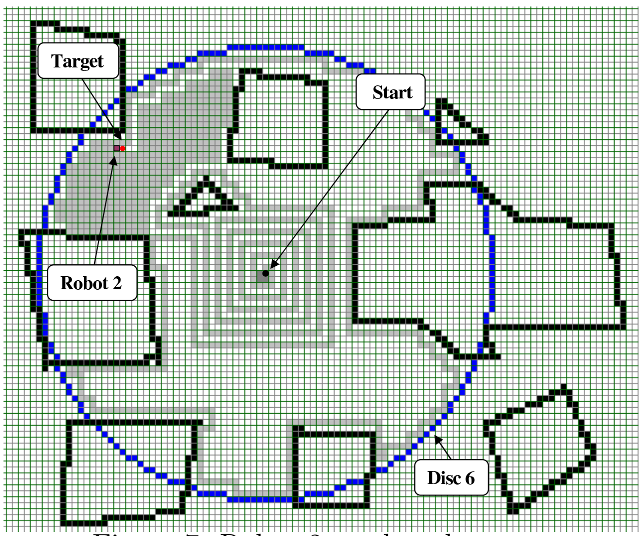

Figure 7: Robot 2 reaches the target

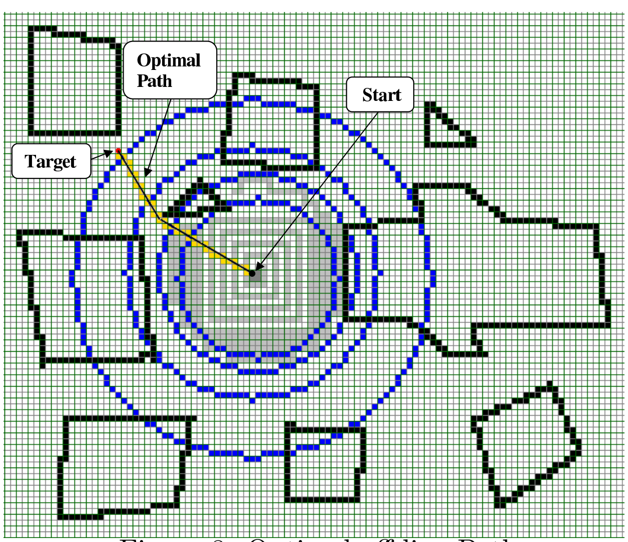

Figure 8: Optimal off-line Path

6 Conclusions

The problem of finding an unknown target in an unknown environment closely relates to many fields in our lives, from supermarkets cleaning, through demining to planetary exploration. The use of multiple robots reduces the time it takes to find the target, and enhances the robustness of the system. Heterogeneity of the robots has many aspects, e.g., different sizes, sensory equipment and velocity. Heterogenous robots groups are sometimes formed in order to benefit from the different capabilities of the robots and sometimes they are formed from combining old and new robot models.

This paper introduces amotion planning algorithm for a group of heterogeneous robots which must find a target whose position is unknown. The environment in which the robots search for the target is unknown, and moreover, it is unbounded, a property missing from most of the current algorithms. The new algorithm is decentralized, deterministic, and its coverage is complete. The robots deployed by the algorithm are using tactile sensors to detect the obstacles in the environment and have very limited or no communication capabilities at all.

The new algorithm's performance is analyzed by finding an upper bound of its performance. The analysis shows that the upper bound is quadratic in the optimal off-line solution, T_{opt}. The new algorithm is tested in simulation.

Future work includes instructing the searching robots to search only in the ring added to the previously covered disc. This is a practical speedup which enhances the performance in the average case. This speedup requires the robots to remember a map of the obstacles encountered in the previously covered discs. Furthermore, a common map between the robots will result in even better performance, but requires constant communication between the robots. Additionally, bounded environments should be taken under consideration, specifically the effect of the robots' size, the initial search disc area, the initial cover time, and the multiplication factor on the upper bound of the algorithm.

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Appendix A

In the following section we present the analytical proof that the upper bound of *H-MRSTM* is quadratic in the optimal off-line solution.

Theorem 1 *(Quadratic competitive complexity)* *Assume target* T *is reachable from* S. *Let* H-MRSTM *use* n *robots with velocities:* $v_i = \beta_i v,\ \forall i,\ i = 1, 2 \ldots n$, $\beta_{i+1} \geq \beta_i \geq 1,\ \forall i,\ i = 1, 2 \ldots n-1$. *Then the traveling time of robot* j *which found the target:*

$$T_{R_j} < \frac{\pi \alpha_j \beta_n^2 v \left(\prod_{i=1}^n \alpha_i\right)}{\beta_j D \left(\prod_{i=1}^n \alpha_i - 1\right)} T_{opt}^2,$$

$$and,\ \ \alpha_i = \frac{(n+1)^{(\frac{1}{n})} \beta_i^{\frac{n-1}{n}}}{\prod_{k=2, k\neq i}^n \beta_k^{\frac{1}{n}}}.$$

Proof: The target is assumed to lie within a disc whose area is $A_{opt} = \pi l_{opt}^2$, where l_{opt} is the optimal off-line path length to the target. First, we will inspect the case which robot no. 1, R_1 finds the target. In worst case scenario, the last robot, R_n, covered a disc whose area is $A_{opt} - \epsilon$ and thus did not find target. Consequently, R_1 is assigned afterwards to search for it in a disc whose area is $\pi \alpha_1 l_{opt}^2$ and finds the target in it with covering time of $T_{i(R_1)} \leq \frac{\pi \alpha_1 l_{opt}^2}{D \beta_1 v}$. The sum of the search times by R_1 is, $T_{R_1} \leq T_1 + T_{1+n} + T_{1+2n} + \cdots + T_i = T_0 + \left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{1+n-j}{n}\right)} T_0 + \left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{1+2n-j}{n}\right)} T_0 + \cdots + \left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{i-j}{n}\right)} T_0 = T_0 + \left(\prod_{i=1}^n \alpha_i\right)^1 T_0 + \left(\prod_{i=1}^n \alpha_i\right)^2 T_0 + \cdots + \left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{i-1}{n}\right)} T_0$

The series of time is a converging geometric series, which its sum is,

$$T_{R_1} \leq \frac{\left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{i-1}{n}+1\right)} - 1}{\prod_{i=1}^n \alpha_i - 1} T_0 < \frac{\left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{i-1}{n}+1\right)}}{\prod_{i=1}^n \alpha_i - 1} T_0 \tag{1}$$

Comparing the two expressions for the time to cover the last disc, $T_{i(R_1)}$, T_0 can be presented in terms of i, $T_{i(R_1)} = \frac{\pi \alpha_1 l_{opt}^2}{\beta_1 D v} = \left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{i-1}{n}\right)} T_0$,

$$T_0 = \frac{\pi \alpha_1 l_{opt}^2}{\beta_1 D v \left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{i-1}{n}\right)}}. \tag{2}$$

Substituting T_0 from (2) into (1) yields, $T_{R_1} < \frac{\left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{i-1}{n}+1\right)}}{\prod_{i=1}^n \alpha_i - 1} \frac{\pi \alpha_1 l_{opt}^2}{\beta_1 D v \left(\prod_{i=1}^n \alpha_i\right)^{\left(\frac{i-1}{n}\right)}}$.

Simplification yields, $T_{R_1} < \frac{\pi \alpha_1 l_{opt}^2 \left(\prod_{i=1}^n \alpha_i\right)}{\beta_1 D v \left(\prod_{i=1}^n \alpha_i - 1\right)}$.

Accordingly, for R_2, in worst case scenario, R_1 covered a disc whose area is $A_{opt} - \epsilon$ and thus did not find target. Consequently, R_2 is assigned afterwards to search for it in a disc whose area is $\pi \alpha_2 l_{opt}^2$ and finds the target in it with covering time of $T_{i(R_2)} \leq \frac{\pi \alpha_2 l_{opt}^2}{D \beta_2 v}$. The sum of the search times by R_2 is,
$T_{R_2} < \frac{\pi \alpha_2 l_{opt}^2 \left(\prod_{i=1}^n \alpha_i\right)}{\beta_2 D v \left(\prod_{i=1}^n \alpha_i - 1\right)}$.

Accordingly, for R_3, in worst case scenario, R_2 covered a disc whose area is $A_{opt} - \epsilon$ and thus did not find target. Consequently, R_3 is assigned afterwards to search for it in a disc whose area is $\pi \alpha_3 l_{opt}^2$ and finds the target in it with covering time of $T_{i(R_3)} \leq \frac{\pi \alpha_3 l_{opt}^2}{D \beta_3 v}$. The sum of the search times by R_3 is,
$T_{R_3} < \frac{\pi \alpha_3 l_{opt}^2 \left(\prod_{i=1}^n \alpha_i\right)}{\beta_3 D v \left(\prod_{i=1}^n \alpha_i - 1\right)}$.

Accordingly, generally, $T_{R_j} < \frac{\pi \alpha_j l_{opt}^2 \left(\prod_{i=1}^n \alpha_i\right)}{\beta_j D v \left(\prod_{i=1}^n \alpha_i - 1\right)}$.

Next, in order to find the optimal multiplication factors, $\alpha_i' s$, a new objective function which combines all the sums of times is formed, $T_{tot} = T_{R_1} + T_{R_2} + \ldots + T_{R_n} < \frac{\pi \alpha_1 l_{opt}^2 \left(\prod_{i=1}^n \alpha_i\right)}{\beta_1 D v \left(\prod_{i=1}^n \alpha_i - 1\right)} + \frac{\pi \alpha_2 l_{opt}^2 \left(\prod_{i=1}^n \alpha_i\right)}{\beta_2 D v \left(\prod_{i=1}^n \alpha_i - 1\right)} + \ldots\ldots + \frac{\pi \alpha_n l_{opt}^2 \left(\prod_{i=1}^n \alpha_i\right)}{\beta_n D v \left(\prod_{i=1}^n \alpha_i - 1\right)} = \frac{\pi l_{opt}^2 \left(\prod_{i=1}^n \alpha_i\right)}{D v \left(\prod_{i=1}^n \alpha_i - 1\right)} \sum_{i=1}^n \frac{\alpha_i}{\beta_i}$

Differentiating T_{tot} according to each of the A_is, comparing each function to zero and finding the common roots yields, $\alpha_i = \frac{(n+1)^{(\frac{1}{n})} \beta_i^{\frac{n-1}{n}}}{\prod_{k=2, k\neq i}^n \beta_k^{\frac{1}{n}}}$ ■

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